Research on Braking Force Distribution Strategy of Composite Braking System of Hybrid Electric Vehicle

J. H. Tang¹, K. Y. Wang†*, S. Y. Bei†, & M. M. Sousa‡

¹School of Automotive and Traffic Engineering, Jiangsu University of Technology, Changzhou City, 213001, China, *E-mail: wkuiy@126.com
‡Fluid Mechanics Laboratory, Mechanical Engineering and Aeronautics Department, University of Patras, Patras, Greece

ABSTRACT: Based on the structure models of the composite braking system with friction brake, hydraulic retarder and regenerative braking, the mathematical models of electric machinery, battery, hydraulic retarder, the temperature rise of friction brake and the braking force distribution strategy on hybrid electric vehicle are established. The braking force distribution curve of composite braking system is studied on basis of maximum braking force of the driving wheel. Based on the maximum recovered energy and the minimum thermal recession of friction brake, the braking force distribution strategy of composite braking system is formulated. The simulation models of the composite brake system are established based on the MATLAB/Simulink software. For the small, medium and large braking strength, the simulation analysis on the braking force distribution, the temperature rise of friction brake and the recovered energy is carried out according to the different speeds and the different adhesion coefficients of road. The simulation results show that the braking force distribution strategy can improve the recovery rate of braking energy and the braking stability, and effectively prevent the occurrence of thermal recession. The mathematical models and simulation models proposed are correct and feasible, which provide a theoretical basis for the follow-up study.

KEYWORDS: Automotive engineering; hybrid electric vehicle; composite braking system; braking force distribution.

INTRODUCTION

The composite braking system includes the regenerative braking of electric machinery, the mechanical friction braking and the hydraulic braking of hydraulic retarder, its structure is shown in Figure (1). The targets of friction brake are braking efficiency and braking stability, the target of hydraulic retarder is the performance to resist thermal degradation, and the regenerative braking takes energy recovery rate as target.

As a complex whole, there are some mutual influence and mutual restriction among them in the process of braking. The braking feeling, braking efficiency and braking stability of traditional braking system will be affected. Therefore, it is necessary to study on the braking force distribution strategy among regenerative braking, friction brake and hydraulic retarder.

At present, all over the world, the extensive researches have been carried out on the composite brake system with braking and friction brake or the composite brake system friction brake and hydraulic retarder [1][2][3]. However, the research on the composite brake system with friction braking, regenerative braking and hydraulic retarder is also

DOI: 10.7508/jmerd.2016.02.013
rarely mentioned. In this paper, taking the maximum energy recovery of braking and the minimum temperature rise of friction brake as targets, the braking force distribution strategy of composite brake system is studied on a selected hybrid electric vehicle based on the ECE regulations, the characteristics of electric machinery, the characteristics of battery, the characteristics of hydraulic retarder, and the temperature model of friction brake. The braking force distribution strategy proposed is modeled, simulated and analyzed, which provide a theoretical basis for the follow-up study.

MATHEMATICAL MODEL OF COMPOSITE BRAKING SYSTEM

Mathematical model of electric machinery

The permanent magnet brushless DC motor is taken as the driving electric machinery in the hybrid electric coach. The corresponding characteristic curves of the torque, power and speed are shown in Figure (2).

In the process of braking, if the rotary speed of electric machinery is lower than the rated speed of \( N_e \), the braking torque of electric machinery is the rated torque of \( T_e \). Otherwise, the power of electric machinery is the rated power of \( P_e \). The relationship between the rotary speed of electric machinery and the speed of vehicle is shown as below.

\[
n = \frac{n u_a i_d}{0.377r} \quad (1)
\]

Where \( n \) is the rotary speed of electric machinery, \( u_a \) is main reduction ratio, \( r \) is the radius of wheel. The maximum braking torque of electric machinery is shown as below.

\[
T_{\text{max}} = \begin{cases} 
T_e & n \leq N_e \\
\frac{9550P_e}{n} & n > N_e 
\end{cases} \quad (2)
\]

![Figure (2). Working characteristic curve of electric machinery.](image)

When the rotary speed of electric machinery is too low, because the back electromotive force of armature is reduced, the electric machinery can’t work efficiently, it will cause the severe wobble of vehicle and influence ride performance. Therefore, at low speeds, the regenerative braking should be out of work. At the same time, in order to avoid the sudden change of braking torque, the regenerative braking should adopt the gradual withdrawal mode [4][5]. The influence factor on the rotary speed of electric machinery is expressed as below.

\[
k_m = \begin{cases} 
0 & \omega_a \leq 50 \\
\frac{\omega_a - 50}{50} & 50 < \omega_a \leq 100 \\
1 & \omega_a > 100
\end{cases} \quad (3)
\]

Where \( \omega_a \) is the angular velocity of electric machinery. The safety and performance of the energy storage system must be considered too. When the SOC is relatively small, the battery will be allowed to be charged. When the SOC is relatively high, the regenerative braking of electric machinery should be prohibited in order to avoid excessive charge of battery. The influence factor on the battery is expressed as below.
Research on braking force distribution strategy of composite braking system of hybrid electric vehicle

\[ k_{\text{SOC}} = \begin{cases} 
1 & \text{SOC} \leq 0.8 \\
10(0.9 - \text{SOC}) & 0.8 < \text{SOC} \leq 0.9 \\
0 & 0.9 < \text{SOC} \leq 1 
\end{cases} \]  

(4)

Therefore, at a certain rotary speed, the maximum braking force of electric machinery is as follows.

\[ F_{\text{max}} = \frac{T_{\text{max}}i_kk_{\text{SOC}}}{r\eta_T} \]  

(5)

Where \( \eta_T \) is the efficiency of mechanical transmission, whose value is taken as 0.9.

Mathematical model of battery

The equivalent circuit of battery in the time of charging is shown as Figure (3). In the figure, \( U \) is the charge voltage, \( I \) is the charge current, \( E \) is the electromotive force of battery, and \( R \) is the internal resistance of battery.

![Figure (3). Equivalent circuit of battery.](image)

The charging power of storage battery is as follows.

\[ P_s = UI = \frac{U(U - E)}{R} \]  

(6)

It can be obtained from the above formula.

\[ U^2 - UE - P_sR = 0 \]  

(7)

So, the charge voltage and the charge current are as follows.

\[ U = \frac{E + \sqrt{E^2 + 4P_sR}}{2} \]  

(8)

\[ I = \frac{U - E}{R} = \frac{\sqrt{E^2 + 4P_sR} - E}{2R} \]  

(9)

The calculation formula of SOC is as follows.

\[ \text{SOC} = \text{SOC}_0 + \frac{\eta_{\text{charge}}}{Q_{\text{ALL}}} \int_{t_0}^{t} \frac{Idt}{R} \]  

(10)

Where \( \text{SOC}_0 \) is the initial value of SOC, \( \eta_{\text{charge}} \) is the charging efficiency, and \( Q_{\text{ALL}} \) is the total power of battery.

In order to prevent the damage caused by the large charge current to battery, the charging power of battery should be limited. When the maximum charge current of \( I_{\text{max}} \) is given, the maximum charge power can be determined.

\[ P_{\text{max}} = U \left( \frac{U - E}{R} \right) = I_{\text{max}}^2R - I_{\text{max}}E \]  

(11)

Mathematical model of hydraulic retarder

If the flow loss caused by liquid leakage is not measured, when the unit mass of liquid is recirculated in the hydraulic retarder, the energy obtained from the pump impeller is equal to the sum of the energy consumed by the
liquid in the process of circulation. The energy consumed is mainly including the loss of friction resistance along the path and the loss of liquid flow impact energy [6].

According to the law of conservation of energy in the hydraulic retarder, it can be obtained:

$$H_z = (\sum H_{mc} + \sum H_j) = 0$$  \hspace{1cm} (12)

Where $H_z$ is the theoretical energy which the pump impeller applies on the oil, $\Sigma H_{mc}$ is the loss energy of frictional resistance, $\Sigma H_j$ is the loss energy of liquid flow impact.

According to the Euler formula, the theoretical energy which the pump impeller applies on the oil is as follows.

$$H_z = \frac{\omega R_z}{g} (\omega R_z + v_m \cot \beta_j)$$  \hspace{1cm} (13)

Where $\omega$ is the angular velocity of pump impeller, $R_z$ is the outlet radius of impeller, $v_m$ is the axial velocity of liquid flow, $\beta_j$ is the angle of the liquid flow at the outlet of impeller, $\beta$ is the blade inclination, and $\beta_j = 90 - \beta$. The loss energy of frictional resistance is as follows.

$$\sum H_{mc} = \frac{\xi_{mc}}{2g} \left( \frac{v_m^2}{2g} \right)$$  \hspace{1cm} (14)

Where $\xi_{mc}$ is the loss coefficient of friction resistance, whose value may be from 0.22 to 0.34. The loss energy of liquid flow impact is as follows.

$$\sum H_j = \frac{\xi_j}{2g} \left( (\omega R_z + v_m \cot \beta_j)^2 + (\omega R_z + v_m \cot \beta_j)^2 \right)$$  \hspace{1cm} (15)

Where $\xi_j$ is the loss coefficient of liquid flow impact, whose value may be from 0.8 to 1.4, $R_1$ is the inlet radius of impeller, $\beta_j$ is the angle of the liquid flow at the inlet of impeller, and $\beta_j = 90 - \beta$. It can be obtained as follows from the formula (12), formula (13), formula (14) and formula (15).

$$\frac{\omega R_z}{g} (\omega R_z + v_m \cot \beta_j) = \frac{\xi_j}{2g} \left( (\omega R_z + v_m \cot \beta_j)^2 + (\omega R_z + v_m \cot \beta_j)^2 \right) + \xi_{mc} \frac{v_m^2}{2g}$$  \hspace{1cm} (16)

The axial velocity of liquid flow can be obtained by the formula (16). The circulating flow of oil can be expressed as follows.

$$Q = v_m A_m$$  \hspace{1cm} (17)

Where $A_m$ is the area of cross section of impeller, whose value can be expressed as follows.

$$A_m = 2\pi R_1 b_1$$  \hspace{1cm} (18)

Where $b_1$ is the channel width of liquid flow at the inlet of impeller. According to the theorem of momentum, the braking torque of hydraulic retarder can be expressed as follows.

$$T_y = \rho Q (R_2 v_{z2} - R_1 v_{z1})$$  \hspace{1cm} (19)

Where $\rho$ is the density of working fluid in hydraulic retarder (In this paper, the engine oil of CF-4 is adopted and its density is 860kg/m$^3$), $v_{z1}$ and $v_{z2}$ are the absolute velocities of oil at the inlet and outlet respectively. Their values are as follows.

$$v_{z1} = \omega R_1 + v_m \cot \beta_j$$  \hspace{1cm} (20)

$$v_{z2} = \omega R_2 + v_m \cot \beta_j$$  \hspace{1cm} (21)

Mathematical model of friction brake’s temperature rise

According to the principle of energy conservation in the braking process, it can be known that the kinetic energy and potential energy of hybrid electric coach consumed in the process of braking are the sum of the energy which are
consumed by friction brake, hydraulic retarder, regenerative braking, rolling resistance and air resistance. If the rolling resistance and air resistance are ignored, then it can be obtained as follows.

\[
\frac{1}{2} m (u_i^2 - u_f^2) + mg \frac{i}{\sqrt{1+i^2}} s = (F_e + F_\beta + F_{r\beta}) s
\]  

(22)

Where \( u_i \) is the initial speed of braking, \( u_f \) is the end speed of braking, \( i \) is the road grade, \( s \) is the braking distance, \( F_e \) is the regenerative braking force of electric machinery, \( F_\beta \) is the braking force of hydraulic retarder, \( F_{r\beta} \) is the friction braking force of front wheel and \( F_{r\beta} \) is the friction braking force of rear wheel.

The temperature rise of friction brake is due to the friction between the friction plate and the brake drum. Hypothesis, the friction force between the friction plate and the brake drum is constant in the process of braking, then according to the principle of friction heat, the rate of temperature rise of front wheel’ friction brake can be expressed as follows.

\[
P_{s_1} = \frac{1}{2} F_{s_1} v_f
\]

(23)

Where \( v_f \) is the relative velocity between the friction plate and the brake drum of front wheel. According to the relevant researches, it shows that about 95% of heat quantity produced by friction brake will be absorbed by the brake drum, then the heat absorption’s rate of front wheel’s brake drum can be expressed as follows [7].

\[
P_{s_1} = 0.95 \frac{F_{s_1} v_f}{2}
\]

(24)

Assuming that the initial temperature of friction brake is \( T_0 \), after a period of time, the temperature of brake drum is increased by \( \Delta T \). According to the energy conservation of brake drum’s temperature rise, it can be obtained as follows.

\[
m_{c_g} c_g \Delta T = P_{s_1} \Delta t
\]

(25)

Where \( m_{c_g} \) is the mass of brake drum, \( c_g \) is the specific heat capacity of brake drum. From the above two equations, the temperature rise model of the front wheel’s friction brake is as follows.

\[
\Delta T_{s_1} = \frac{0.95 F_{s_1} v_f \Delta t}{2 m_{c_g}}
\]

(26)

In the same way, it can obtained that the temperature rise model of the rear wheel’s friction brake is as follows.

\[
\Delta T_{s_2} = \frac{0.95 F_{s_2} v_f \Delta t}{2 m_{c_g}}
\]

(27)

Where \( v_f \) is the relative velocity between the friction plate and the brake drum of rear wheel.

In the course of downhill braking with constant velocity, the friction brake not only has the temperature rise, but also has the temperature decline. The main reason for the decrease of temperature is the cooling of the brake drum. According to the theory of thermodynamics, the heat dissipation of the brake drum includes three ways which are heat conduction, heat convection and heat radiation. Among them, the heat dissipating capacities of heat conduction and heat radiation are smaller, so the effect of heat convection on the brake drum is only considered here. According to the Newton cooling formula, the heat flow rate of heat convection on brake drum is as follows.

\[
P_{s_2} = h_a A (T_f - T_a)
\]

(28)

Where \( h_a \) is the coefficient of convective heat transfer between the brake drum and the air, \( A \) is the outer surface area of brake drum, \( T_f \) is the temperature of front wheel’s brake drum, and \( T_a \) is the air temperature around brake drum. From the relevant experimental data, it can be known that the function relationship between the coefficient of convective heat transfer and the speed of coach is as follows [8].

\[
h_a = 5.224 + 1.5525 u_i e^{-0.02778 u_i}
\]

(29)

It can be obtained as follows from the formula (28) and formula (29).
\[ P_s = (5.224 + 1.5525u_e e^{-0.0027785u_e}) A(T_f - T_i) \]  

(30)

The energy conservation equation of brake drum cooling is as follows.

\[ m_e c_g \Delta T = -P_s \Delta t \]  

(31)

From the above two formulas, the cooling model of front wheel’s friction brake is as follows.

\[ \Delta T_{fr} = \frac{-\left(5.224 + 1.5525u_e e^{-0.0027785u_e}\right) A(T_f - T_i) \Delta t}{m_e c_g} \]  

(32)

In the same way, the cooling model of rear wheel’s friction brake is as follows.

\[ \Delta T_{rd} = \frac{-\left(5.224 + 1.5525u_e e^{-0.0027785u_e}\right) A(T_r - T_i) \Delta t}{m_e c_g} \]  

(33)

Where \( T_i \) is the temperature of rear wheel’s brake drum. Therefore, combining the temperature rise and the cooling, the integrated temperature rise model of front wheel’s friction brake is as follows.

\[ \Delta T_{fr} = \Delta T_{fr} + \Delta T_{fd} = \frac{0.95 F_{mv} V \Delta t}{2m_e c_g} - \frac{-\left(5.224 + 1.5525u_e e^{-0.0027785u_e}\right) A(T_f - T_i) \Delta t}{m_e c_g} \]  

(34)

And the integrated temperature rise model of rear wheel’s friction brake is as follows.

\[ \Delta T_{rd} = \Delta T_{rd} + \Delta T_{rd} = \frac{0.95 F_{mv} V \Delta t}{2m_e c_g} - \frac{-\left(5.224 + 1.5525u_e e^{-0.0027785u_e}\right) A(T_r - T_i) \Delta t}{m_e c_g} \]  

(35)

Though the integral calculation to the above two formulas, the current temperature of friction brake drums of front wheel and rear wheel can obtain.

BRAKING FORCE DISTRIBUTION STRATEGY OF COMPOSITE BRAKING SYSTEM

In this article, the driving shaft of hybrid electric vehicle selected is the rear axle, whose braking force is provided by the friction brake, regenerative braking and hydraulic braking. Through increasing the braking force of rear axle, the utilization ratio of regenerative braking force may be improved and the thermal recession of friction brake may be reduced. However, the excessive braking force of rear axle will cause that the rear wheel will be locked firstly, and the braking stability will be affected. Therefore, the reasonable braking force distribution strategy can improve the utilization rate of regenerative braking, reduce the thermal recession of friction brake, and ameliorate the braking stability of vehicle.

Constraint requirements of the ECE regulations

In order to ensure the stability of braking and the sufficient attachment efficiency, the braking regulations of R13 ECE was established. It was be defined in the ECE regulations that the constraint formula of braking strength is \( z \geq 0.1 + 0.85 (\varphi - 0.2) \) when the adhesion coefficient of road is in the range from 0.2 to 0.8. It is that the constraint formula on the utilization attachment coefficient of front and rear axles is \( \varphi \leq \left( z + 0.07 \right) / 0.85 \).

The braking requirements of the ECE regulations for the M3 type of coach are shown as Figure (4). When the brake strength is between 0.15 and 0.3, the utilization adhesion coefficient curve of front axle should be above the utilization adhesion coefficient curve of rear axle. The utilization adhesion coefficient of front axle is between the curve of \( \varphi = z + 0.08 \) and the curve of \( \varphi = z - 0.08 \). The utilization adhesion coefficient of rear axle is less than or equal to \( z + 0.08 \). When the brake strength is greater than or equal to 0.3, the constraint formula of utilization adhesion coefficient of rear axle is \( z \geq 0.3 + 0.74 (\varphi - 0.38) \).
According to the boundaries of braking stability are determined by the ECE regulations, the relation formulas of the braking forces of front and rear axles are as follows.

\[
\begin{align*}
F_{ab1} &= \frac{(z + 0.07)G}{0.85} \left( \frac{a - z_h}{L} \right) \\
F_{ab2} &= Gz - F_{ab2}
\end{align*}
\]  
(36)

Where \( F_{ab1} \) is the ground braking force of front wheel, \( F_{ab2} \) is the ground braking force of rear wheel, \( G \) is the gravity of vehicle, \( L \) is the wheelbase of vehicle, \( h_g \) is the centroid height of vehicle, \( a \) is the distance between the center of mass and the front axle, \( b \) is the distance between the center of mass and the rear axle. The maximum braking force of front axle and rear axle can be obtained by the formula (36) when the braking stability and the efficiency of braking are ensured.

Control structure of composite braking system

The control structure of composite braking system is as shown in Figure (5). According to the adhesion coefficient of road, the braking intensity, the ECE regulations and the real-time speed, the controller of composite brake system determines the allocation proportion of braking force between the front and rear axles, and distributes the target braking force to friction brake, electric machinery and hydraulic retarder. The friction brake, electric machinery and hydraulic retarder work according to the instructions of brake controller, and achieve the function of deceleration or braking.

Braking force distribution strategy of composite braking system

In the process of braking, the braking force of composite braking system can be provided by friction brake, hydraulic retarder or electric machinery. However, the targets of friction brake are braking efficiency and braking stability, the target of hydraulic retarder is the performance to resist thermal degradation, and the regenerative braking takes energy recovery rate as target. Therefore, on the basis of ensuring the safety performance of braking, the economic performance of braking should be optimized. The safety performance of braking is reflected in the following aspects: (1) There will be no rear or front wheel locked, ensuring the stability of braking; (2) It should be avoided that the temperature of friction brake is too high and the phenomenon of thermal recession appears. And the economic performance of braking is reflected by the maximum of braking energy feedback. In order to meet the above requirements, the target braking force should be assigned preferentially to regenerative braking, secondly to
hydraulic retarder and finally to friction brake. Therefore, the braking force distribution strategy of composite braking system between the front axle and the rear axle is established as follows.

(1) When the adhesion coefficient of road is smaller (\(\varphi \leq 0.329\)).

Assuming that the road adhesion coefficient is 0.2, the braking force distribution curve is shown as Figure (6).

![Figure (6). The braking force distribution curve of \(\varphi=0.2\).](image)

When the brake strength is smaller, the full braking force is provided with rear wheel (as shown in the OA section). When the braking intensity is bigger, the braking force is provided by the front and rear wheels simultaneously (as shown in the AC section). The line of AC is parallel to the line of r when the adhesion coefficient of road is 0.2, and is under the line of r. There is a certain distance between the line of AC and the line of r, the purpose is to ensure that the rear wheel will not be locked firstly according to the distribution strategy of braking force. When the braking intensity is greater than the braking intensity of the point C, the target braking force is distributed according to the ideal braking force distribution curve of I.

Under the different braking strengths, the maximum braking force of rear wheel is determined as follows.

\[
F_{\text{max}} = \begin{cases} 
Gz & \text{if } z \leq z_A \\
\varphi \left( G - (a + z_h) \right) - \psi & \text{if } z_A < z \leq z_C \\
Gz & \text{if } z_C < z \leq \varphi \\
\psi & \text{if } z > \varphi 
\end{cases}
\]  

(37)

Where \(z_A\) is the brake strength of point A, \(z_C\) is the brake strength of point C, \(\psi\) is the adjustment margin.

(2) When the adhesion coefficient of road is bigger (\(\varphi > 0.329\)).

Assuming that the road adhesion coefficient is 0.6, the braking force distribution curve is shown as Figure (7).

![Figure (7). The braking force distribution curve of \(\varphi=0.6\).](image)
When the brake strength is smaller, the full braking force is provided with rear wheel (as shown in the OA section). When the braking intensity is bigger, the braking force is provided by the front and rear wheels simultaneously (as shown in the AB section and BC section). The curve of AB is parallel to the curve of the ECE regulations, and is under the curve. There is a certain distance between them, the purpose is to ensure that the braking force distribution strategy can meet the requirements of ECE regulations. The line of BC is parallel to the line of $r$ when the adhesion coefficient of road is 0.6, and is under the line of $r$. There is a certain distance between the line of BC and the line of $r$, the purpose is to ensure that the rear wheel will not be locked firstly according to the distribution strategy of braking force. When the braking intensity is greater than the braking intensity of the point C, the target braking force is distributed according to the ideal braking force distribution curve of I.

Under the different braking strengths, the maximum braking force of rear wheel is determined as follows.

$$F_{br\text{max}} = \begin{cases} Gz & z \leq z_a \\ \frac{z + 0.07 G}{0.85} (a - zh) - \psi & z_a < z \leq z_b \\ \frac{\varphi G}{L} (a - zh) - \psi & z_b < z \leq z_c \\ \frac{Gz}{L} (a - zh) & z_c < z \leq \varphi \\ \frac{G}{L} (a - \varphi h) & z > \varphi \end{cases}$$  \hfill (38)

Where $z_a$ is the brake strength of point B.

Braking force distribution strategy of rear axle

When the braking strength is $z$, the expectation braking force of hybrid electric vehicle is as follows.

$$F = Gz = F_{\mu 1} + F_{\mu 2} = F_{\mu 1} + F_{b 2} = F_{\gamma} + F_{r}$$  \hfill (39)

Where $F_{\mu 1}$ is the braking force of front axle, $F_{\mu 2}$ is the braking force of rear axle, $F_{\gamma}$ is the braking force of hydraulic retarder, and $F_{r}$ is the braking force of electric machinery. According to the adhesion coefficient of road, the maximum braking force of rear axle can be obtained by the equation (37) or equation (38).

The braking force of front axle is the difference between the expectation braking force and the braking force of rear axle. The braking force of rear axle is priority provided by the regenerative braking force. If the maximum regenerative braking force is less than the braking force of rear axle, the rest of braking force is provided by hydraulic retarder. If the rest of braking force is greater than the maximum braking force of hydraulic retarder, the friction brake of rear axle begins to work. Thus, the braking force distribution strategy of rear axle is as follows.

1. If $F_{\text{max}} \geq F_{\text{brmax}}$, then
   $$F_x = F_{\text{brmax}}, F_y = 0, F_{\mu 1} = 0, F_{\mu 2} = 0$$  \hfill (40)

2. If $F_{\text{max}} < F_{\text{brmax}}$ and $(F - F_x) \leq F_{\text{ymax}}$, then
   $$F_x = F_{\text{max}}, F_y = F_{\text{brmax}} - F_x, F_{\mu 1} = F - F_{\text{brmax}}, F_{\mu 2} = 0$$  \hfill (41)

3. If $F_{\text{max}} < F_{\text{brmax}}$ and $(F - F_x) > F_{\text{ymax}}$, then
   $$F_x = F_{\text{max}}, F_y = F_{\text{ymax}}, F_{\mu 1} = F - F_{\text{brmax}}, F_{\mu 2} = F_{\text{brmax}} - F_x - F_y$$  \hfill (42)

Where $F_{\text{ymax}}$ is the maximum braking force of hydraulic retarder.

SIMULATION MODELING AND ANALYSIS

Establishment of simulation models

According to the above mathematical models, the simulation models of electric machinery, battery, hydraulic retarder, temperature rise of friction brake, braking force distribution strategy are established based on MATLAB/Simulink software. The overall simulation model of the composite braking system is shown in the following diagram.
Simulation results and analysis

The change relationship curves of braking force distribution, braking energy, recycling energy, temperature rise of friction brake with braking time are analyzed when the adhesion coefficient of road selected is 0.2 and 0.6, the initial velocity is 40km/h and 80km/h, the brake strength is 0.2, 0.5 and 0.7 respectively.

(1) Braking with small strength (such as $z=0.2$).

When the braking strength is small, the simulation results are shown in Figure (9), Figure (10) and Figure (11).
Figure (11). Relationship curves of braking energy and recycling energy with time.

Through the simulation analysis, it can be known that the distribution of braking force and the temperature rise of friction brake have nothing to do with the adhesion coefficient of road. This is because the braking force distribution of front and rear wheels is only related to the braking strength at this point. This is in agreement with the braking force distribution strategy formulated.

From the Figure (9), it can be known that the braking force of front axle is zero, the braking force of rear axle is mainly undertaken by regenerative braking and hydraulic retarder. When the braking forces of electric machinery and hydraulic retarder reduces with the decrease of revolution speed of wheel, the residual braking force is provided by the friction brake. From the Figure (10), it can be known that if the initial speed is higher, the temperature rise of friction brake is higher too. Hypothesis, the braking force of rear wheel is completely borne by the friction brake, then the maximum temperature of friction brake will reach 140 °C, which is increased by 120 °C. Therefore, the braking force distribution strategy of composite braking system can effectively reduce the thermal recession. From the Figure (11), it can be known that the recycling energy is 167.3J and 444.7J respectively when the initial speed is 40km/h and 80km/h. But the recovery rate of braking energy is 19% and 11% respectively. This is because the braking time is longer with the raise of the initial speed. In the braking process, the longer the working time of electric machinery, the more the regenerative braking energy. Therefore, the number of recycling energy has a direct relationship with the initial speed, the braking force of electric machinery and the time of regenerative braking, and the recovery rate of braking energy has a direct relationship with braking energy.

(2) Braking with medium strength (such as \(z=0.5\)).

When the braking strength is medium and the initial speed is 40km/h, the simulation results of the braking force distribution, the temperature rise, the braking energy and the recycling energy are shown in Figure (12), Figure (13) and Figure (14).

Figure (12). Relationship curve between braking force and time.
Research on braking force distribution strategy of composite braking system of hybrid electric vehicle

From the Figure (12), Figure (13) and Figure (14), it can be known that the braking force distribution, the temperature rise of friction brake, the braking energy and the recycling energy are different on the roads with different adhesion coefficients when the brake strength is moderate. Compared to the road of low adhesion coefficient, the braking force of rear axle is significantly greater than the braking force of front axle when the vehicle is braked on the road of high adhesion coefficient. This is because the braking force is assigned in accordance with the ideal braking force distribution curve on the road of low adhesion coefficient, and the braking force is assigned in accordance with the curve of the ECE regulations on the road of high adhesion coefficient. From the Figure (13) and Figure (14), it can be known that the bigger the braking force of rear axle, the higher the temperature rise of rear wheel’s friction brake. But, because the electric machinery has been working in the area of maximum value when the adhesion coefficient is 0.2 and 0.6 respectively, the recovered braking energies are same for the two kinds of adhesion coefficient.

(3) Braking with large strength (such as $z=0.7$).

When the braking strength is large and the initial speed is 40km/h, the simulation results of the braking force distribution, the temperature rise, the braking energy and the recycling energy are shown in Figure (15), Figure (16) and Figure (17).
From the Figure (15) and Figure (16), it can be known that the curves of the distribution of braking force and the temperature rise are similar to the medium braking strength. The braking force distribution, the temperature rise of friction brake, the braking energy and the recycling energy are same on the roads with different adhesion coefficients. From the Figure (11), Figure (14) and Figure (16), it can be known that the recovered energy of electric machinery is reduced with the increase of the braking strength at a certain initial speed. This is because the regenerative braking force which electric machinery can provide has been close to the maximum in the small braking strength, and the braking time is decreased significantly with the increase of braking strength, which results in the decrease of recovered energy.

CONCLUSION
On the basis of ensuring the braking stability and safety, taking the maximum recovered braking energy and the minimal thermal depression as targets, the braking force distribution strategy of composite braking system is formulated on hybrid electric vehicle. The simulation models of composite braking system are established based on MATLAB/Simulink software. The following conclusions are drawn by analysis.

(1) The braking force distribution strategy of composite braking system proposed is correct and feasible. The maximum recovered braking energy can be achieved in the small, medium and large braking strength. At a certain initial speed, the recovered braking energy is reduced with the increase of the braking strength. And at a certain braking strength, the recovered braking energy is increased with the raise of initial speed.

(2) The temperature rise of friction brake of hybrid electric vehicle with hydraulic retarder is obviously decreased, relative to the temperature rise of traditional friction brake. The higher the initial speed, the greater the braking strength, and the more obvious temperature rise. So then the performance of resisting thermal degradation of friction brake is effectively improved.

CONFLICT OF INTEREST
The authors confirm that this article content has no conflict of interest.

ACKNOWLEDGEMENTS
This research is supported by the Natural Science Foundation of Jiangsu Province (Grants No. BK2012586), the University Science Research Project of Jiangsu Province (13KJB580005), the Jiangsu Province Ordinary University Graduate Research and Innovation Project (CXZZ13_0659).

REFERENCES

Figure (16). Relationship curves of braking energy and recycling energy with time.
Research on braking force distribution strategy of composite braking system of hybrid electric vehicle


